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Stochastic Modeling on Mixture Distribution with Application to Using Cancer Survival Data

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ABSTRACT

In this paper, specific statistical considerations are typically required, in order to select the best model for fitting survival data. The proposed the new mixture of Gamma and Shanker Distribution (MGSD), so named because it specifically mixes of two distributions: Shanker and gamma. There is also Reliability Analysis, statistical features such as stochastic ordering, moments, order statistics, entropy, and the Maximum Likelihood Estimation of the model parameters estimating. Lastly, a two real cancer data set is used to demonstrate the use of the AIC, BIC, and AICC model selection methods. It is compared with the fit and show that the (MGS) distribution is more flexible than the other distributions.

Keyword: Mixture distribution; Maximum likelihood Estimation.

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1. Introduction

Medical research is mostly interested in studying the survival of cancer patients, as applied to statistical research. The statistical distributions have been extensively utilized for analyzing time-to-event data, also referred to as survival or reliability data, in different areas of applicability, including medical science. In recent years, an impressive set of new statistical distributions has been explored by statisticians. The necessity of developing an extended class of classical distribution is analysis, biomedicine, reliability, insurance, and finance. Recently, many researchers have been working in this area and have proposed new methods to develop improved probability distributions with utility. A statistical study is frequently used, which extensively depends on the presumptive probability model or distributions.

A Shanker distribution RV X with a parameter $\theta > 0$ is described by its pdf is defined as

$$f(x,\theta) = \frac{\theta^2}{\theta^2+1}(\theta+x)e^{-\theta x} \quad ,x>0,\theta>0$$

Considering the gamma distribution with parameters $\lambda = 3, \theta$ the pdf can be defined as

$$f(\theta,x) = \frac{1}{2}\theta^3 x^2\, e^{-\theta x} \quad ,x>0,\theta>0$$

It has introduced by Lindley [6], the fiducial distribution and bayes theorem. Rama Shanker [16], has been introduced a mixture of exponential (θ) and gamma (2, θ) distribution proposed a shanker distribution. Shanker [13], the combination of exponential (θ), gamma (2, θ) and gamma (4, θ) with respectively mixing proportion $\frac{\theta^3}{\theta^3+\theta^2+6}, \frac{\theta^2}{\theta^3+\theta^2+6}$ *and* $\frac{6}{\theta^3+\theta^2+6}$ proposed Uma distribution. Akash distribution is a two-component mixture of an exponential distribution and gamma distribution with their mixing proportions $\frac{\theta^2}{\theta^2+2}$ and $\frac{2}{\theta^2+2}$, Shanker [18]. Proposed a Komal distribution with applications in survival analysis, Ramma Shanker [21], the combination of exponential (θ) and gamma (2, θ) distribution with mixing proportions $\frac{\theta(\theta+1)}{\theta^2+\theta+1}$ and $\frac{1}{\theta^2+\theta+1}$.

This article creates the (MGSD) that was proposed using a blend of Shanker and Gamma distributions. The probability density function (pdf) and cumulative distribution function (cdf) of the proposed distribution, together with a few of its statistical characteristics and a reliability analysis, have been generated and given for the rest of this research work. An approach to the maximum likelihood estimation for estimating the model parameters. Lastly, the results of fitting the other well-known distributions to the cancer survival data using (MGSD) likewise demonstrate this. All computations in this research were performed using the statistical programming language R.

2. New Mixture Distribution

This section introduces the MGS distribution, which is a new distribution created by combining two existing distributions. Let X be a random variable with a mixed distribution. Its density function (pdf), f(x), is expressed as follows

$$f(x) = \sum_{i=1}^{k} p_i f_i(x)$$

$f_i(x)$probability density function for all I, p_i $i = 1, \dots, n$ denote mixing proportions that are non-negative and $\sum_{i=1}^{k} p_i = 1$. The $f_1(x) \sim$ gamma (3, θ) with parameters $\lambda = 3$, θ and $f_2(x) \sim$ Shanker(θ) with parameter θ two independents random variables with $\frac{\lambda^2}{\lambda+1}$ and $\frac{1}{\lambda+1}$respectively. Now the density function of X is given by

$$f(x;\lambda,\theta) = \frac{\theta^2}{\lambda+1}\left(\frac{\lambda\theta}{2}x^2 + \frac{\theta+x}{(\theta^2+1)}\right)e^{-\theta x} \quad ,x > 0, \theta > 0, > 0 \qquad (1)$$

The function defined in (1) represents a probability distribution function $f(x;\lambda,\theta)\ for\ all\ x > 0,$

$$\int_0^\infty f(x;\lambda,\theta)\,dx = \int_0^\infty \frac{\theta^2}{\lambda+1}\left(\frac{\lambda\theta}{2}\,x^2 + \frac{(\theta+x)}{\theta^2+1}\right)e^{-\theta x}dx$$

$$= \frac{\theta}{\lambda+1}\left(\frac{\lambda+1}{\theta}\right)$$

$$= 1$$

The cumulative distribution function cdf is defined as

$$F(x;\lambda,\theta) = \int_0^x \frac{\theta^2}{\lambda+1}\left(\frac{\lambda\theta}{2}\,x^2 + \frac{\theta+x}{(\theta^2+1)}\right)e^{-\theta x}dt$$

$$F(x;\lambda,\theta) = \frac{1}{\lambda+1}\left[1 + \left(1 - \left(\left(\frac{\theta x}{2}+1\right)\theta x + 1\right)\lambda + \frac{\theta x}{\theta^2+1} + 1\right)e^{-\theta x}\right] x > 0, \theta, \lambda > 0 \qquad (2)$$

The function meets the requirements for a cumulative distribution function.

$$\lim_{x\to 0} F(x;\lambda,\theta) = \lim_{x\to 0}\left[\frac{1}{\lambda+1}\left[1 + \left(1 - \left(\left(\frac{\theta x}{2}+1\right)\theta x + 1\right)\lambda + \frac{\theta x}{\theta^2+1} + 1\right)e^{-\theta x}\right]\right]$$

$$= 1 - 1 = 0$$

$$\lim_{x\to\infty} F(x;\lambda,\theta) = \lim_{x\to\infty}\left[\frac{1}{\lambda+1}\left[1+\left(1-\left(\left(\frac{\theta x}{2}+1\right)\theta x+1\right)\lambda+\frac{\theta x}{\theta^2+1}+1\right)e^{-\theta x}\right]\right]$$
$$= 1 - 0 = 1$$

3. Reliability Analysis

This section will provide the reliability function, hazard function, reverse hazard function, cumulative hazard function, Odds rate, Mills ratio and, Mean Residual function for the specified MGS distribution.

3.1 Survival Function

The survival function of MGS distribution is defined as

$$S(x) = 1 - F(x;\lambda,\theta)$$
$$S(x) = 1 - \frac{1}{\lambda+1}\left[1+\left(1-\left(\left(\frac{\theta x}{2}+1\right)\theta x+1\right)\lambda+\frac{\theta x}{\theta^2+1}+1\right)e^{-\theta x}\right]$$

3.2 Hazard Rate Function

An important metric for describing life phenomena is the hazard rate function of the MGS distribution, which is defined by $h(x) = \frac{f(x;\lambda,\theta)}{\mathbf{1}-F(x;\lambda,\theta)}$.

$$h(x) = \frac{\theta^2\left(\frac{\lambda\theta x^2}{2}+\frac{\theta+x}{(\theta^2+1)}\right)}{(\lambda+1)-\left[1+\left(1-\left(\left(\frac{\theta x}{2}+1\right)\theta x+1\right)\lambda+\frac{\theta x}{\theta^2+1}+1\right)e^{-\theta x}\right]}$$

3.3 Revers hazard rate

The Revers hazard rate of MGS distribution is defined as

$$h_r(x) = \frac{f(x;\lambda,\theta)}{F(x;\lambda,\theta)}$$
$$h_r(x) = -\frac{\theta^2\left(\frac{\lambda\theta x^2}{2}+\frac{\theta+x}{(\theta^2+1)}\right)}{\left[1+\left(1-\left(\left(\frac{\theta x}{2}+1\right)\theta x+1\right)\lambda+\frac{\theta x}{\theta^2+1}+1\right)e^{-\theta x}\right]}$$

3.4 Cumulative hazard function

The Cumulative hazard function of MGS distribution is defined as

$$H(x) = -\ln(1 - F(x;\lambda,\theta))$$
$$H(x) = \ln\left[\frac{1}{\lambda+1}\left[1+\left(1-\left(\left(\frac{\theta x}{2}+1\right)\theta x+1\right)\lambda+\frac{\theta x}{\theta^2+1}+1\right)e^{-\theta x}\right]-1\right]$$

3.5 Odds rate function

The Odds rate function of MGS distribution is defined as

$$O(x) = \frac{F(x;\lambda,\theta)}{1-F(x;\lambda,\theta)}$$
$$O(x) = \frac{\left[1+\left(1-\left(\left(\frac{\theta x}{2}+1\right)\theta x+1\right)\lambda+\frac{\theta x}{\theta^2+1}+1\right)\right]}{(\lambda+1)-\left[1+\left(1-\left(\left(\frac{\theta x}{2}+1\right)\theta x+1\right)\lambda+\frac{\theta x}{\theta^2+1}+1\right)\right]}$$

3.6 Mean Residual function

The mean residual function of MGS distribution is defined as

$$M(x) = \frac{1}{S(x)}\int_x^\infty t\, f(t)dt - x$$
$$M(x) = \frac{1}{1-\frac{1}{\lambda+1}\left[1+\left(1-\left(\left(\frac{\theta x}{2}+1\right)\theta x+1\right)\lambda+\frac{\theta x}{\theta^2+1}+1\right)e^{-\theta x}\right]}\int_x^\infty t\ \frac{\theta^2}{\lambda+1}\left(\frac{\lambda\theta}{2}t^2+\frac{\theta+t}{(\theta^2+1)}\right)e^{-\theta t}dt - x$$

Then, integration using the substitution method

$$Let,\ \theta t = x,\ \ t = \frac{x}{\theta}\ \ and\ \ \ dt = \frac{1}{\theta}dx$$

Then, $t = x, \quad x = \theta x, \quad and\ x \to \infty, \quad t \to \infty$
After solving the integral,
Then, using the following upper incomplete gamma function is defined as
$\int_x^{\infty} x^{z-1} e^{-z}\, dz = \Gamma(z, x)$

$$M(x) = \left(\frac{\left(\frac{\lambda\Gamma(4,\theta x)}{2} + \frac{\theta^2\Gamma(2,\theta x)+\Gamma(3,\theta x)}{(\theta^2+1)}\right)}{\theta\left[(\lambda+1) - \left[1 + \left(1 - \left(\left(\frac{\theta x}{2}+1\right)\theta x+1\right)\lambda + \frac{\theta x}{\theta^2+1}+1\right)e^{-\theta x}\right]\right]} \right) - x$$

4. Statistical Properties

In this section, we derived the structural properties, moments, the moment generating function, Characteristic function and r^th moment for the MGS distribution of the random variable. Including, the Mean, Variance, Coefficient of Variation, Skewness, Kurtosis, and, Dispersion investigated.

4.1 Moments

The r^th moments of a RV X, is defined as

$$E(X^r) = \mu_r' = \int_0^{\infty} x^r\ f(x;\lambda,\theta)\, dx$$

$$\mu_r' = \int_0^{\infty} x^r\ \frac{\theta^2}{\lambda+1}\left(\frac{\lambda\theta}{2}x^2\ + \frac{\theta+x}{(\theta^2+1)}\right) e^{-\theta x} dx$$

$$\mu_r' = \frac{\theta^2}{\lambda+1}\int_0^{\infty} x^r\ \left(\frac{\lambda\theta}{2}\ x^2 + \frac{(\theta+x)}{\theta^2+1}\right) e^{-\theta x}\ dx$$

Simplifying the integration,
Then, using the following gamma function is defined as

$$\int_0^{\infty} x^{z-1} e^{-px}\, dx = \frac{\Gamma(z)}{p^z}$$

$$\mu_r' = \left[\frac{\lambda\Gamma(r+3)}{2\theta^r(\lambda+1)} + \frac{\left(\theta^2\Gamma(r+1)+\Gamma(r+2)\right)}{\theta^r(\lambda+1)(\theta^2+1)}\right] \tag{3}$$

Where, $\Gamma(.)$ Is the gamma function. Subsequently, the mean and variance can be defined by substituting $r = 1,2,3,4$ in equation (3),

$$E(X) = mean = \left[\frac{3\lambda(\theta^2+1)+(\theta^2+2)}{\theta(\lambda+1)(\theta^2+1)}\right]$$

$$E(X^2) = \left[\frac{12\lambda(\theta^2+1)+2(\theta^2+6)}{\theta^2(\lambda+1)(\theta^2+1)}\right]$$

$$E(X^3) = \left[\frac{60\lambda(\theta^2+1)+6(\theta^2+4)}{\theta^3(\lambda+1)(\theta^2+1)}\right]$$

$$E(X^4) = \left[\frac{360\lambda(\theta^2+1)+2(12\theta^2+60)}{\theta^4(\lambda+1)(\theta^2+1)}\right]$$

Variance $= \sigma^2 = E(X^2) - \left(E(X)\right)^2$

$$\sigma^2 = \left[\frac{12\lambda}{\theta^2(\lambda+1)} + \frac{2(\theta^2+6)}{\theta^2(\lambda+1)(\theta^2+1)}\right] - \left[\frac{3\lambda}{\theta(\lambda+1)} + \frac{(\theta^2+2)}{\theta(\lambda+1)(\theta^2+1)}\right]^2$$

Simplification, also obtained as

$$\sigma^2 = \frac{12\lambda\theta^4+48\lambda^2\theta^2+48\lambda^2+\theta^4+4\theta^2+6\lambda+2-\lambda\theta^4}{\theta^2(\lambda+1)^2(\theta^2+1)^2}$$

$$\sigma = \frac{\sqrt{12\lambda\theta^4+48\lambda^2\theta^2+48\lambda^2+\theta^4+4\theta^2+6v+2-\lambda\theta^4}}{\theta(\lambda+1)(\theta^2+1)}$$

Coefficient of Variation

$$C.V\left(\frac{\sigma}{\mu}\right)=\frac{\sqrt{12\lambda\theta^4+48\lambda^2\theta^2+48\lambda^2+\theta^4+4\theta^2+6\lambda+2-\lambda\theta^4}}{3\lambda(\lambda+1)(\theta^2+2)}$$

Skewness

$$Sk(X)=\sqrt{\beta_1}=\frac{E(X^3)}{(var(X))^{\frac{3}{2}}}$$

After simplification,

$$Sk(X)=\frac{\left(60\lambda(\theta^2+1)+6(\theta^2+4)\right)[(\lambda+1)(\theta^2+1)]^2}{(12\lambda\theta^4+48\lambda^2\theta^2+48\lambda^2+\theta^4+4\theta^2+6\lambda+2-\lambda\theta^4)^{\frac{3}{2}}}$$

Kurtosis

$$Ku(X)=\beta_1=\frac{E(X^4)}{(var(X))^2}$$

$$Ku(X)=\frac{\left(360\lambda(\theta^2+1)+2(12\theta^2+60)\right)\left((\lambda+1)(\theta^2+1)\right)^3}{(12\lambda\theta^4+48\lambda^2\theta^2+48\lambda^2+\theta^4+4\theta^2+6\lambda+2-\lambda\theta^4)^2}$$

Dispersion

Dispersion$=\frac{\sigma^2}{\mu}$

$$=\frac{12\lambda\theta^4+48\lambda^2\theta^2+48\lambda^2+\theta^4+4\theta^2+6\lambda+2-\lambda\theta^4}{\left(\theta(\lambda+1)(\theta^2+1)\right)\left(3\lambda(\theta^2+1)+(\theta^2+2)\right)}$$

Simplification, and also obtained as

$$=\frac{12\lambda\theta^4+48\lambda^2\theta^2+48\lambda^2+\theta^4+4\theta^2+6\lambda+2-\lambda\theta^4}{(3\lambda+1)\lambda\theta^3(\theta^2+1)+(2\theta^3+2)\lambda(\theta+1)+2\theta(\lambda+1)(\theta^2+1)}$$

4.2 Moment Generating Function

The MGF of a RV X is denoted by $M_X(t)$ and is defined as

$$M_X(t)=E(e^{tx})=\int_0^\infty e^{tx}\,f(x;\lambda,\theta)\,dx \quad ,t\epsilon\mathcal{R}$$

$$M_X(t)=\frac{\theta^2}{\lambda+1}\int_0^\infty e^{tx}\left(\frac{\lambda\theta}{2}x^2+\frac{(\theta+x)}{\theta^2+1}\right)e^{-\theta x}\,dx$$

$$M_X(t)=\frac{\theta^2}{\lambda+1}\left[\frac{\lambda\theta}{2}\int_0^\infty x^2e^{-(\theta-t)x}\,dx+\frac{1}{\theta^2+1}\left(\theta\int_0^\infty e^{-(\theta-t)x}\,dx+\int_0^\infty x\,e^{-(\theta-t)x}\,dx\right)\right]$$

Then, using integration by substitution method

$$u=x^2\quad and\quad dv=-\frac{e^{-(\theta-t)x}}{(\theta-t)}$$

Substitute the limits of integration and simplify the expression

$$M_X(t)=\frac{\theta^2}{(\lambda+1)(\theta-t)^2}\left[\frac{\lambda\theta}{(\theta-t)}+\frac{\theta(\theta-t)+1}{\theta^2+1}\right] \tag{4}$$

The characteristics function (CF) of a RV X, it is denoted by $\phi_X(t)$ and is defined as

$$\phi_X(t)=E\left(e^{itx}\right)=\int_0^\infty e^{itx}\,f(x;\lambda,\theta)\,dx$$

$$\phi_X(t)=M_X(it)$$

$$\phi_X(t)=\frac{\theta^2}{(\lambda+1)(\theta-it)^2}\left[\frac{\lambda\theta}{(\theta-it)}+\frac{\theta(\theta-it)+1}{\theta^2+1}\right] \tag{5}$$

5. Hormonic Mean

If H_X is the harmonic mean (HM) of the RV X, then

$$H_X=E\left[\frac{1}{X}\right]$$

$$H.M=\int_0^\infty\frac{1}{x}\,\frac{\theta^2}{\lambda+1}\left(\frac{\lambda\theta}{2}x^2+\frac{\theta+x}{(\theta^2+1)}\right)e^{-\theta x}dx$$

So, the final result of the integral is

$$H.M = \frac{\theta^2}{\lambda+1}\left(\frac{\lambda}{\theta} + \frac{2}{\theta^2(\theta^2+1)}\right)$$

6. Mean

The Mean deviation (MD) of the RV X, is defined as

$$D(\mu) = E(|X-\mu|)$$

$$D(\mu) = \int_0^{\infty} |X-\mu| \; f(x;\lambda,\theta)dx$$

$$D(\mu) = 2\mu F(\mu) - 2\int_0^{\mu} x \; f(x;\lambda,\theta)dx$$

Then,

$$\int_0^{\mu} x\, f(x;\lambda,\theta)\, dx = \int_0^{\mu} x \;\frac{\theta^2}{\lambda+1}\left(\frac{\lambda\theta}{2}x^2 \; + \frac{\theta+x}{(\theta^2+1)}\right) e^{-\theta x}\, dx$$

Using the integration by substitution method

Let, $\theta x = t,\; x = \frac{t}{\theta},\; and \quad dx = \frac{1}{\theta}dt$

$Then, x = 0, \qquad t = 0, \qquad and\;\; x = \mu, \qquad t = \theta\mu$

Then, using the following lower incomplete gamma function is defined as

$$\gamma(s,x) = \int_0^{x} t^{s-1}e^{-t}dt$$

Substitute the limits of integration and simplify the expression.

$$D(\mu) = 2\left[\frac{3\lambda(\theta^2+1)+(\theta^2+2)}{\theta(\lambda+1)(\theta^2+1)}\right]\frac{1}{\lambda+1}\left[1 \;+\left(1-\left(\left(\frac{\theta\mu}{2}+1\right)\theta\mu+1\right)\lambda+\frac{\theta\mu}{\theta^2+1}+1\right)e^{-\theta\mu}\right] - \frac{2}{\theta(\lambda+1)}\left(\frac{\lambda\gamma(4,\theta\mu)}{2}+\frac{\theta^2\gamma(2,\theta\mu)+\gamma(3,\theta\mu)}{(\theta^2+1)}\right)$$

7. Median

The mean deviation from median of the RV X, is defined as

$$D(M) = E(|X-M|)$$

$$D(M) = \int_0^{\infty} |X-M| \; f(x;\lambda,\theta)dx$$

$$D(M) = \int_0^{M}(M-x)f(x;\lambda,\theta)dx + \int_M^{\infty}(x-M)f(x;\lambda,\theta)dx$$

Then, simplification

$$D(M) = \mu - 2\int_0^{M} x\, f(x;\lambda,\theta)dx$$

Now,

$$\int_0^{M} x\, f(x;\lambda,\theta)dx = \int_0^{M} x \;\frac{\theta^2}{\lambda+1}\left(\frac{\lambda\theta}{2}x^2 \; + \frac{\theta+x}{(\theta^2+1)}\right) e^{-\theta x}\, dx$$

Then, using integration by substitution method

Let, $\theta x = t,\; x = \frac{t}{\theta},\; and \quad dx = \frac{1}{\theta}dt$

$Then, x = 0, \qquad t = 0, \qquad and\;\; x = M, \qquad t = \theta M$

So, the final expression for the integral is

$$D(M) = \left[\frac{3\lambda(\theta^2+1)+(\theta^2+2)}{\theta(\lambda+1)(\theta^2+1)}\right] - \frac{2}{\theta(\lambda+1)}\left(\frac{\lambda\gamma(4,\theta M)}{2}+\frac{\theta^2\gamma(2,\theta M)+\gamma(3,\theta M)}{(\theta^2+1)}\right)$$

8. Order Statistics

The derived pdf of the i^{th} order statistics of the MGS distribution. Let $X_1, X_2, \dots, X_n$ be a simple random sample from mixture of shanker and gamma distribution with cdf and pdf given by (1) and (2), respectively. Let $X_{(1:n)} \leq X_{(2:n)} \leq \cdots \leq X_{(n:n)}$ denote the order statistics defined from this sample. We now given the pdf of $X_{r:n}$, say $f_{r;n}(x)$ of $X_{r:n}$, $i = 1,2,\dots,n$. The pdf of the r^{th} order statistics $X_{r:n}$, $r = 1,2,\dots,n$ is defined as

$$f_{r:n}(x) = \frac{n!}{(r-1)!(n-r)!}\left(F(x)\right)^{r-1}\left(1-F(x)\right)^{n-r} f(x), x > 0 \quad (6)$$

Where $F(.)$ and $f(.)$ are given by (1) and (2) respectively,

$$and \;\; W_{\;r:n} = \frac{n!}{(r-1)!(n-r)!}$$

$$f_{r:n} = W_{\;r:n}\left(F(x)\right)^{r-1}\left(1-F(x)\right)^{n-r} f(x)$$

Then, using the following binomial expansion

$$(1-z)^a = \sum_{j=0}^{\infty}(-1)^j \binom{a}{j} z^j$$

$$f_{r:n} = W_{\;r:n}\sum_{s=0}^{\infty}(-1)^s \binom{n-r}{s}\left(F(x)\right)^{r+s-1} f(x) \quad (7)$$

$$f_{r:n} = W_{\;r:n}\frac{\theta^2}{\lambda+1}\sum_{s=0}^{\infty}(-1)^s \binom{n-r}{s}\left[\frac{1}{\lambda+1}\left[1+\left(1-\left(\left(\frac{\theta x}{2}+1\right)\theta x+1\right)\lambda+\frac{\theta x}{\theta^2+1}+1\right)e^{-\theta x}\right]\right]^{r+s-1}\left(\frac{\lambda\theta}{2}x^2+\frac{(\theta+x)}{\theta^2+1}\right)e^{-\theta x}$$

Simplification

$$f_{r:n} = W_{\;r:n}\frac{\theta^2}{\lambda+1}\sum_{s=0}^{\infty}\sum_{t=0}^{\infty}\left(\frac{1}{\lambda+1}\right)^{r+s-1}(-1)^s\binom{n-r}{s}\binom{r+s-1}{t}\left[\lambda\left(1-\left(\left(\frac{\theta x}{2}+1\right)\theta x+1\right)\right)+\frac{\theta x}{\theta^2+1}+1\right]^t e^{-\theta(t+1)x}\left(\frac{\lambda\theta x^2}{2}+\frac{(\theta+x)}{\theta^2+1}\right)$$

First order statistics

$$f_{1:n} = W_{\;1:n}\frac{\theta^2}{\lambda+1}\sum_{s=0}^{\infty}\sum_{t=0}^{\infty}\left(\frac{1}{\lambda+1}\right)^{s}(-1)^s\binom{n-1}{s}\binom{s}{t}\left[\lambda\left(1-\left(\left(\frac{\theta x}{2}+1\right)\theta x+1\right)\right)+\frac{\theta x}{\theta^2+1}+1\right]^t e^{-\theta(t+1)x}\left(\frac{\lambda\theta}{2}x^2+\frac{(\theta+x)}{\theta^2+1}\right)$$

n^{th} order statistics

$$f_{n:n} = W_{\;n:n}\frac{\theta^2}{\lambda+1}\sum_{t=0}^{\infty}\left(\frac{1}{\lambda+1}\right)^{n+s-1}(-1)^s\binom{n+s-1}{t}\left[\lambda\left(1-\left(\left(\frac{\theta x}{2}+1\right)\theta x+1\right)\right)+\frac{\theta x}{\theta^2+1}+1\right]^t e^{-\theta(t+1)x}\left(\frac{\lambda\theta x^2}{2}+\frac{(\theta+x)}{\theta^2+1}\right)$$

8.1 Quantile function

The quantile function of a distribution with cdf, $F(x;\lambda,\theta)$, is defined by $q = F\left(x_q;\lambda,\theta\right)$, where $0 < q < 1$. Thus, the quantile function of MSG distribution is given by

$$1-q = \frac{1}{\lambda+1}\left[1+\left(1-\left(\left(\frac{\theta x_q}{2}+1\right)\theta x_q+1\right)\lambda+\frac{\theta x_q}{\theta^2+1}+1\right)e^{-\theta x_q}\right]$$

Figure 8 show the quantile plot for different values of $\theta \; and \; \lambda$.

9. Entropies

In this section, we derived the Rényi entropy, and Tsallis entropy from the MGS distribution.

It is well known that entropy and information can be considered measures of uncertainty or the randomness of a probability distribution. It is applied in many fields, such as engineering, finance, information theory, and biomedicine. The entropy functionals for probability distribution were derived on the basis of a variational definition of uncertainty measure.

9.1 Rényi Entropy

Entropy is defined as a random variable X is a measure of the variation of the uncertainty. It is used in many fields, such as engineering, statistical mechanics, finance, information theory, biomedicine, and economics. The entropy measure is the Rényi of order which is defined as

$$R_\gamma = \frac{1}{1-\gamma}\log\int_0^\infty [f(x;\ \lambda,\theta)]^\gamma\ dx \qquad ;\gamma > 0, \gamma \neq 1$$

$$R_\gamma = \frac{1}{1-\gamma}\log\int_0^\infty \left[\frac{\theta^2}{\lambda+1}\left(\frac{\lambda\theta}{2}x^2 + \frac{(\theta+x)}{\theta^2+1}\right)e^{-\theta x}\right]^\gamma\ dx$$

Using the following Binomial series expansion is defined as

$$(a+b)^z = \sum_{j=0}^\infty \binom{Z}{j}\ (a)^j b^{z-j}$$

Then, the following power series expansion is defined as

$$\alpha^x = \sum_{k=0}^\infty \frac{(x\ln\alpha)^k}{k!}$$

Substitute the expansion and Simplification also obtained as

$$R_\gamma = \frac{1}{1-\gamma}\log\left(\frac{\theta^2}{\lambda+1}\right)^\gamma \sum_{j=0}^\infty\sum_{k=o}^\infty\sum_{l=0}^\infty \left(\frac{\lambda\theta}{2}\right)^j\left(\frac{1}{\theta^2+1}\right)^{\gamma-j}\binom{\lambda}{j}\binom{\lambda-j}{k}\frac{(k\ln(\theta))^l}{l!}\int_0^\infty x^{\gamma+j-k}\ e^{-\gamma\theta x}\ dx$$

Solving the integral, using the following gamma function is defined as

$$\int_0^\infty x^{z-1}e^{-px}\,dx = \frac{\Gamma(z)}{p^z}$$

So, the final expression for the integral is

$$R_\gamma = \frac{1}{1-\gamma}\log\left(\frac{\theta^2}{\gamma+1}\right)^\gamma \sum_{j=0}^\infty\sum_{k=o}^\infty\sum_{l=0}^\infty \left(\frac{\lambda\theta}{2}\right)^j\left(\frac{1}{\theta^2+1}\right)^{\gamma-j}\binom{\lambda}{j}\binom{\lambda-j}{k}\frac{(k\ln(\theta))^l}{l!}\frac{\Gamma(\gamma+j-k+1)}{(\gamma\theta)^{\gamma+j-k+1}}$$

9.2 Tsallis Entropy

The Boltzmann-Gibbs (B-G) statistical properties initiated by Tsallis have received a great deal of attention. This generalization of (B-G) statistics was first proposed by introducing the mathematical expression of Tsallis entropy (Tsallis, (1988) for continuous random variables, which is defined as

$$T_\gamma = \frac{1}{\gamma-1}\left[1-\int_0^\infty [f(x;\ \lambda,\theta)]^\gamma\ dx\right] \qquad ;\gamma > 0, \gamma \neq 1$$

$$T_\gamma = \frac{1}{\gamma-1}\left[1-\int_0^\infty \left(\frac{\theta^2}{\lambda+1}\right)^\gamma\left(\left(\frac{\lambda\theta}{2}x^2 + \frac{(\theta+x)}{\theta^2+1}\right)e^{-\theta x}\right)^\gamma\ dx\right]$$

Then, substitute the limits of integration and the final expressions for the integral is

$$T_\gamma = \frac{1}{1-\delta}\left(1-\left(\frac{\theta^2}{\gamma+1}\right)^\gamma \sum_{j=0}^\infty\sum_{k=o}^\infty\sum_{l=0}^\infty \left(\frac{\lambda\theta}{2}\right)^j\left(\frac{1}{\theta^2+1}\right)^{\gamma-j}\binom{\lambda}{j}\binom{\lambda-j}{k}\frac{(k\ln(\theta))^l}{l!}\frac{\Gamma(\gamma+j-k+1)}{(\gamma\theta)^{\gamma+j-k+1}}\right)$$

10. Stochastic Ordering

A crucial technique in reliability and finance for evaluating the relative performance of the models is stochastic ordering. Let X and Y be two random variables with pdf, cdf, and reliability functions $f(x), f(y), F(x), F(y). S(x) = 1 - F(x) and\ F(y)$.

1- Likelihood ratio order ($X \leq_{LR}\ Y$) if $\frac{f_X(x;\ \lambda,\theta)}{f_Y(x;\ \lambda,\theta)}$ decreases in x

2- Stochastic order ($X \leq_{ST}\ Y$) if $F_X(x;\ \lambda,\theta) \geq F_Y(x;\ \lambda,\theta)$ for all x

3- Hazard rate order ($X \leq_{HR} Y$) if $h_X(x;\ \lambda,\theta) \geq F_Y(x;\ \lambda,\theta)$ for all x
4- Mean residual life order ($X \leq_{MRL} Y$) if $MRL_X(X) \leq MRL_Y(X)$ for all x

Prove that the MSG distribution complies with the ordering with the highest likelihood (the likelihood ratio ordering).

Assume that X and Y are two independent Random variables with probability distribution function$f_x(x;\lambda,\theta)$and $f_y(x;\beta,\alpha)$If $\lambda < \alpha$ and $\theta < \beta$, then

$$\Lambda = \frac{f_x(x;\lambda,\theta)}{f_y(x;\beta,\lambda)}$$

$$\Lambda = \frac{\frac{\theta^2}{\lambda+1}\left(\frac{\lambda\theta}{2}x^2 + \frac{(\theta+x)}{\theta^2+1}\right)e^{-\theta x}}{\frac{\alpha^2}{\beta+1}\left(\frac{\beta\alpha}{2}x^2 + \frac{(\alpha+x)}{\alpha^2+1}\right)e^{-\alpha x}}$$

$$\Lambda = \frac{\theta^2(\beta+1)\left(\frac{\lambda\theta}{2}x^2 + \frac{(\theta+x)}{\theta^2+1}\right)}{\lambda^2(\lambda+1)\left(\frac{\beta\alpha}{2}x^2+\frac{(\alpha+x)}{\alpha^2+1}\right)}e^{-(\theta-\alpha)x}$$

Therefore,

$$\log[\Lambda] = \log\left[\frac{\theta^2(\beta+1)}{\alpha^2(\lambda+1)}\right] + \log\left[\frac{\lambda\theta x^2}{2} + \frac{(\theta+x)}{\theta^2+1}\right] - \log\left[\frac{\beta\alpha x^2}{2} + \frac{(\alpha+x)}{\alpha^2+1}\right] - (\theta-\alpha)x$$

Differentiating with respect to x, also obtained as.

$$\frac{\partial\log[\Lambda]}{\partial x} = \frac{\left[\lambda\theta x+\frac{1}{\theta^2+1}\right]}{\left[\frac{\lambda\theta}{2}x^2 + \frac{(\theta+x)}{\theta^2+1}\right]} - \frac{\left[\beta\alpha x+\frac{1}{\alpha^2+1}\right]}{\left[\frac{\beta\alpha}{2}x^2 + \frac{(\alpha+x)}{\alpha^2+1}\right]} + (\theta-\lambda)$$

Hence, $\frac{\partial\log[\Lambda]}{\partial x} < 0$ if $\lambda < \beta$, $\theta < \alpha$.

11. Bonferroni and Lorenz Curves

The Bonferroni and Lorenz curves have been obtained using the MGS distribution in this section. The Bonferroni and Lorenz curve is a powerful tool in the analysis of distributions and has applications in many fields, such as economies, insurance, income, reliability, and medicine. The Bonferroni and Lorenz cures for a X be the random variable of a unit and $f(x)$ be the probability density function of x. $f(x)dx$ will be represented by the probability that a unit selected at random is defined as

$B(p) = \frac{1}{p\mu}\int_0^q x\ (x;\lambda,\theta)dx$ and

$$L(p) = \frac{1}{\mu}\int_0^q x\ f(x;\lambda,\theta)dx$$

Where, $q = F^{-1}(p);\quad q\epsilon[0,1]$ and $\mu = E(X)$

Hence the Bonferroni and Lorenz curves of our distribution are, given by

$$\mu = \left[\frac{3\lambda(\theta^2+1)+(\theta^2+2)}{\theta(\lambda+1)(\theta^2+1)}\right]$$

$$B(p) = \frac{1}{p\mu}\int_0^q x\ \frac{\theta^2}{\lambda+1}\left(\frac{\lambda\theta}{2}x^2 + \frac{\theta+x}{(\theta^2+1)}\right)e^{-\theta x}\ dx$$

$$B(p) = \frac{\theta^2}{p(\lambda+1)}\left(\frac{\lambda\theta}{2}\int_0^q x^3 e^{-\theta x}\ dx + \frac{1}{(\theta^2+1)}\left(\theta\int_0^q x\ e^{-\theta x}\ dx + \int_0^q x^2\ e^{-\theta x}\ dx\right)\right)$$

Then, integration using the substitution method

Let, $\theta x = t,\ x = \frac{t}{\theta},\ and\quad dx = \frac{1}{\theta}dt$

$Then, x = 0,\qquad t = 0,\qquad and\ \ x = q,\qquad t = \theta q$

Substitute the limits of integration and simplify the final expression is

$$B(p) = \frac{1}{p}\left[\frac{1}{\theta(\lambda+1)}\left(\frac{\lambda\gamma(4,\theta q)}{2} + \frac{\theta^2\gamma(2,\theta q)+\gamma(3,\theta q)}{\theta^2+1}\right)\right]$$

$L(p) = pB(p)$

$L(p) = \left[\frac{1}{\theta(\lambda+1)}\left(\frac{\lambda\gamma(4,\theta q)}{2} + \frac{\theta^2\gamma(2,\theta q)+\gamma(3,\theta q)}{\theta^2+1}\right)\right]$

12. Estimations of Parameter

The MGS distribution parameter's maximum likelihood estimates and Fisher's information matrix are provided in this section.

12.1 Maximum Likelihood estimation (MLE)and Fisher's Information Matrix

Consider $x_1, x_2, x_3, \dots, x_n$ be a random sample of size n from the MGS distribution with parameter α, θ the likelihood function, which is defined as

$L = (x; \lambda, \theta) = \prod_{i=1}^{n} f(x_i; \lambda, \theta)$

$L = \prod_{i=1}^{n} \frac{\theta^2}{\lambda+1}\left(\frac{\lambda\theta}{2}x_i^2 + \frac{\theta+x_i}{(\theta^2+1)}\right)e^{-\theta x_i}$

Then, the log-likelihood function is

$\ell = \log L = n\log(\theta^2) - n\log(\lambda+1) + n\log\sum_{i=1}^{n}\left(\frac{\lambda\theta}{2}x_i^2 + \frac{\theta+x_i}{(\theta^2+1)}\right) - \theta\sum_{i=1}^{n}x_i$

differentiating with respect to θ *and* λ

$$\frac{\partial \log L}{\partial\theta} = \frac{2n}{\theta} + \sum_{i=1}^{n}\frac{\left(\frac{\lambda x_i^2}{2} + \frac{(1-\theta^2-2\theta x_i)}{(\theta^2+1)^2}\right)}{\left(\frac{\lambda\theta}{2}x_i^2 + \frac{\theta+x_i}{(\theta^2+1)}\right)} - \sum_{i=1}^{n}x_i = 0 \quad (8)$$

$$\frac{\partial \log L}{\partial\lambda} = -\frac{n}{(\lambda+1)} + \sum_{i=1}^{n}\frac{\left(\frac{\theta x_i^2}{2}\right)}{\left(\frac{\lambda\theta}{2}x_i^2 + \frac{(\theta+x_i)}{\theta^2+1}\right)} = 0 \quad (9)$$

The maximum likelihood estimate of the parameters for the MGS distribution is provided by equations (8) and (9). The equation, however, cannot be solved analytically, so we used R programming and a data set to solve it numerically.

The asymptotic normality results are used to derive the confidence interval. Given that if $\hat{\lambda} = (\hat{\theta}, \hat{\lambda})$ represents the MLE of $\lambda = (\theta, \lambda)$, the results can be expressed as follows:

$\sqrt{n}(\hat{\lambda} - \lambda) \to N_2(0, I^{-1}(\lambda))$

In this case, $I(\lambda)$represents Fisher's Information Matrix.

$$I(\lambda) = -\frac{1}{n}\begin{pmatrix} E\left[\frac{\partial^2 \log L}{\partial\theta^2}\right] & E\left[\frac{\partial^2 \log L}{\partial\theta\partial\lambda}\right] \\ E\left[\frac{\partial^2 \log L}{\partial\lambda\partial\theta}\right] & E\left[\frac{\partial^2 \log L}{\partial\lambda^2}\right] \end{pmatrix}$$

$$\left[\frac{\partial^2 \log L}{\partial\theta^2}\right] = \frac{2n}{\theta^2} + \sum_{i=1}^{n}\frac{\left(\frac{\gamma\theta}{2}x_i^2 + \frac{\theta+x_i}{(\theta^2+1)}\right)\left(\frac{6\theta^4-2\theta^5-4(1-2x_i)\theta^3-4(x_i-2)\theta^2-2(1-4x_i)\theta-2x_i}{(\theta^2+1)^4}\right) - \left(\frac{\gamma x_i^2}{2} + \frac{(1-\theta^2-2\theta x_i)}{(\theta^2+1)^2}\right)^2}{\left(\frac{\lambda\theta x_i^2}{2} + \frac{\theta+x_i}{(\theta^2+1)}\right)^2}$$

$$\left[\frac{\partial^2 \log L}{\partial\lambda^2}\right] = \frac{n}{(\lambda+1)^2} - \sum_{i=1}^{n}\frac{\theta^2 x_i^2}{2\left(\frac{\lambda\theta x_i^2}{2} + \frac{\theta+x_i}{(\theta^2+1)}\right)^2}$$

$$\left[\frac{\partial^2 \log L}{\partial\theta\partial\lambda}\right] = \sum_{i=1}^{n}\frac{\frac{x_i^2}{4}\left(\lambda\theta x_i^2 + \frac{\theta+x_i}{(\theta^2+1)}\right) - \left(\frac{\lambda\theta^2 x_i^2 - \theta x_i + 2\theta^2 x_i^3}{4(\theta^2+1)^2}\right)}{\left(\frac{\lambda\theta}{2}x_i^2 + \frac{\theta+x_i}{(\theta^2+1)}\right)^2}$$

13. Applications

Dat set 1: This data includes the life expectancy (in years) of forty patients with leukemia, a blood malignancy, from one of Saudi Arabia's Ministry of Health facilities, as published in [3]. This real information is

0.315, 0.496, 0.616, 1.145, 1.208, 1.263, 1.414, 2.025, 2.036, 2.162, 2.211, 2.370, 2.532, 2.693, 2.805, 2.910, 2.912, 3.192, 3.263, 3.348, 3.427, 3.499, 3.534, 3.767, 3.751, 3.858, 3.986, 4.049, 4.244, 4.323, 4.381, 4.392, 4.397, 4.647, 4.753, 4.929, 4.973, 5.074, 5.381.

Data set 2: The data under consideration are the life times of 20 leukemia patients who were treated by a certain drug [4]. The data are

1.013, 1.034, 1.109, 1.226, 1.509, 1.533, 1.563, 1.716, 1.929, 1.965, 2.061, 2.344, 2.546, 2.626, 2.778, 2.951, 3.413, 4.118, 5.136.

Data set 3: The dataset included the survival rates of 121 breast cancer patients with were treated at a major hospital to 1929 to 1938 (Lee, 1992). (A1-kadim and Mahdi, 2018) [2] has recently used this dataset.

0.3, 0.3, 4.0, 5.0, 5.6, 6.2, 6.3, 6.6, 6.8, 7.4, 7.5, 8.4, 8.4, 10.3,11.0, 11.8, 12.2, 12.3, 13.5, 14.4, 14.4, 14.8, 15.5, 15.7, 16.2, 16.3, 16.5, 16.8, 17.2, 17.3, 17.5,17.9, 19.8, 20.4, 20.9, 21.0, 21.0, 21.1, 23.0, 23.4, 23.6, 24.0, 24.0, 27.9, 28.2, 29.1, 30.0, 31.0,31.0, 32.0, 35.0, 35.0, 37.0, 37.0, 37.0, 38.0, 38.0, 38.0, 39.0, 39.0, 40.0, 40.0, 40.0, 41.0, 41.0,41.0, 42.0, 43.0, 43.0, 43.0, 44.0, 45.0, 45.0, 46.0, 46.0, 47.0, 48.0, 49.0, 51.0, 51.0, 51.0, 52.0,54.0, 55.0, 56.0, 57.0, 58.0, 59.0, 60.0, 60.0, 60.0, 61.0, 62.0, 65.0, 65.0, 67.0, 67.0, 68.0, 69.0, 78.0, 80.0, 83.0, 88.0, 89.0, 90.0, 93.0, 96.0, 103.0, 105.0, 109.0, 109.0, 111.0, 115.0, 117.0, 125.0, 126.0, 127.0, 129.0, 129.0, 139.0, 154.0.

The Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), Akaike Information Criteria Corrected (AICC), and $-2\log L$.are used to compare the goodness of fit of the fitted distribution.

The following formula can be used to determine AIC, BIC, AICC, and $-2\log L$..

$$AIC = 2k - 2\log L\,,\; BIC = k\log n - 2\log L \; and \; AICC = AIC + \frac{2k(k+1)}{(n-k-1)}$$

Where, $k =$ number of parameters, n sample size and $-2\log L$ is the maximized value of loglikelihood function.

A basic statistical description of the dataset is given in Table 1. Figures (9) indicate that Q-Q and P-P plots are suitable models for the dataset. The box plots for the data set are shown in Figure (10), (11), and (12).

Table 1. The statistical approach of the cancer patient's dataset.

No	n	Mean	Median	Variance	Std. Deviation	Skewness	Kurtosis
Data set 1	39	3.13541	3.34800	1.894	1.376209	-.416	-.727
Data set 2	19	2.24053	1.96500	1.199	1.094858	1.218	1.444
Data set 3	121	46.173	39.500	1251.968	35.3832	1.068	0.477

Table 2. MLEs of the fitted distribution for the parameters estimated values of cancer datasets.

Data Set. No	Model	$\hat{\theta}$	$\hat{\lambda}$
1	Mixture of gamma and shanker distribution	0.9903102 (0.1214810)	1.7502840 (0.8702718)
	Lindely	0.2577071 (0.06161721)	
	Shanker	0.54972161 (0.05806214)	
	Rama	1.10146523 (0.08055189)	
	Exponential	0.31893857 (0.05107054)	
	Aradhana	0.75060122 (0.07108124)	
	Akash	0.80168363 (0.07120997)	
	Ishita	0.80668240 (0.06521656)	
2	Mixture of gamma and shanker distribution	1.2974573 (0.5114641)	5.8552727 (2.8921143)
	Lindely	0.7076860 (0.1200725)	
	Shanker	0.7124395 (0.10777871)	
	Rama	1.3784229 (0.1415338)	
	Exponential	0.4463246 (0.1023934)	
	Aradhana	0.985545 (0.135948)	
	Akash	0.0297001 (0.1317933)	
	Ishita	0.9975990 (0.1134076)	
3	Mixture of gamma and shanker distribution	0.047916107 (0.002982318)	0.047916107 (0.025017403)
	Lindely	0. 042301604 (0.002718848)	
	Shanker	0.043180645 (0.002771516)	
	Rama	0.086335660 (0.003923506)	
	Exponential	0.021597929 (0.001959228)	
	Aradhana	0.042301604 (0.002718848)	
	Akash	0.064664492 (0.003390847)	
	Ishita	0.064904911 (0.003403705)	

Table 3. Goodness-of-fit Statistics for the Cancer Dataset's

Data Set. No	Model	-2log L	AIC	BIC	AICC
1	Mixture of gamma and shanker distribution	144.0257	148.0257	151.3528	148.3590
	Lindely	156.5028	158.5028	160.1664	158.6080
	Shanker	144.7945	155.9545	157.6181	156.0597
	Rama	143.3158	154.3158	147.1023	154.4210
	Exponential	167.1353	169.1353	170.7988	169.0405
	Aradhana	149.4283	151.4283	153.0918	151.5335
	Akash	149.0561	151.0561	152.7196	151.1613
	Ishita	147.9967	149.9967	151.6603	150.1019
2	Mixture of gamma and shanker distribution	53.59311	57.59311	59.48199	58.34311
	Lindely	64.02158	66.02158	66.96602	66.2438
	Shanker	63.08856	65.08856	66.033	65.3107
	Rama	62.41991	64.41991	65.36435	64.6421
	Exponential	68.65501	70.65501	71.59945	70.8772
	Aradhana	60.60053	62.60053	63.54497	62.8227
	Akash	62.69158	64.69158	65.63602	64.9138
	Ishita	62.74297	64.74297	65.68741	64.9651
3	Mixture of gamma and shanker distribution	1034.789	1038.789	1044.38	1038.8907
	Lindely	1160.863	1162.863	1165.659	1162.8966
	Shanker	1165.784	1167.784	1170.586	1167.8176
	Rama	1241.883	1243.883	1246.679	1243.9166
	Exponential	1170.256	1172.256	1175.051	1172.2896
	Aradhana	1187.828	1162.863	1165.659	1162.8966
	Akash	1193.125	1195.125	1197.121	1195.1586
	Ishita	1201.28	1203.28	1206.076	1203.3136

In comparison to the mixture of gamma and shanker distribution, Lindely, Shanker, Rama, Exponential, Aradhana, Akash, and Ishita distribution it is evident from table 1 and 2's results that the (MGS) distribution has smaller AIC, BIC, and AICC values. This suggests that the mixture of the shanker and gamma distribution fits the data better. Therefore, compared to the other distributions, the (MGS) distribution provides a better fit.

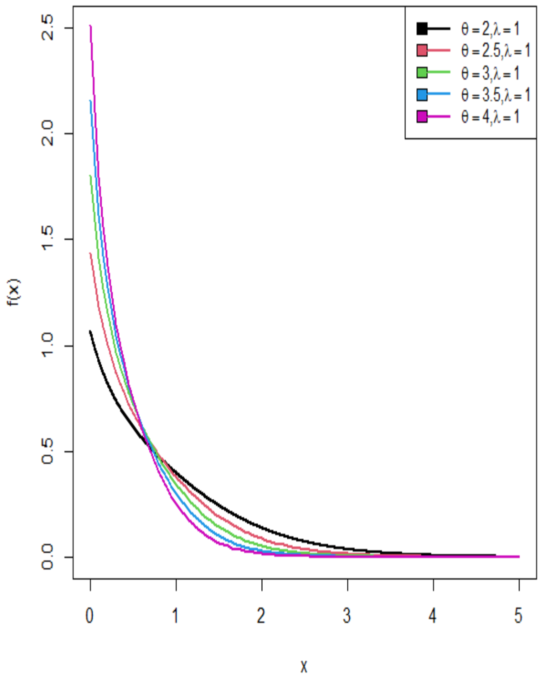


Figure.1:Pdf plot of the Mixture of gamma and shanker distribution

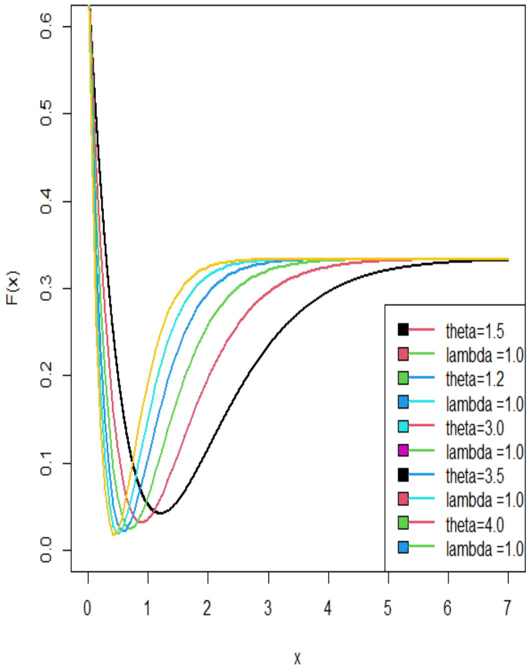


Figure.2 cdf plot of the mixture of gamma and shanker distribution

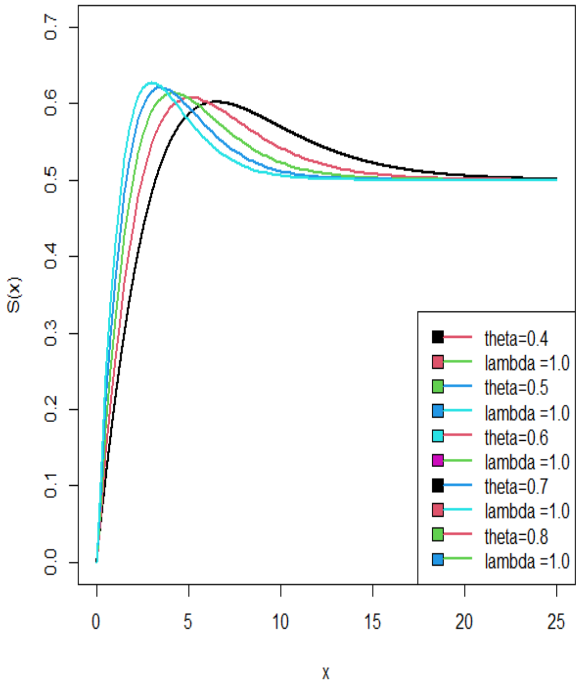
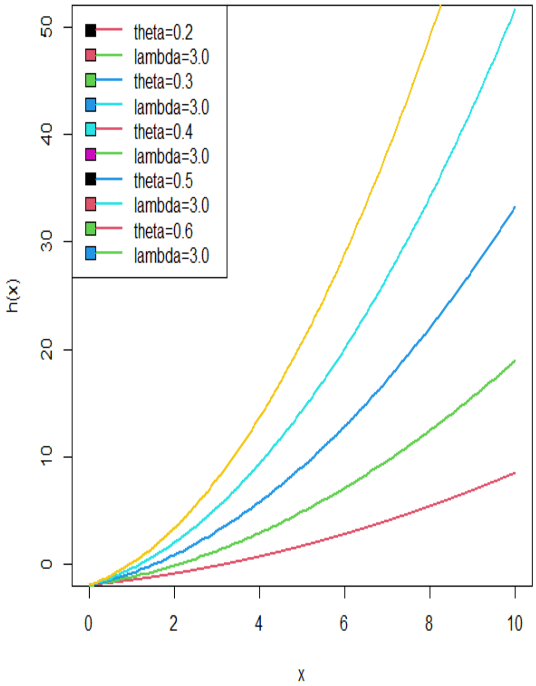
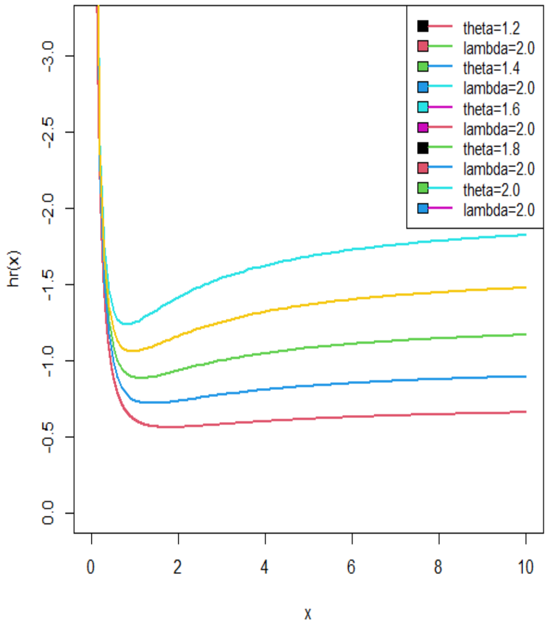


Figure.3 Survival plot of the Mixture of gamma and shanker distribution



Figures.4 Hazard plot of the Mixture of gamma and shanker distribution



Figures.5 Revers hazard plot of the Mixture of gamma and shanker distribution

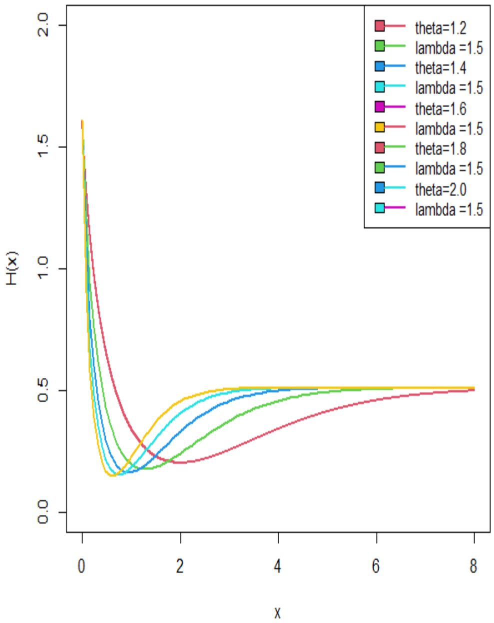


Figure.6 Cumulative hazard plot of the Mixture of gamma and shanker distribution

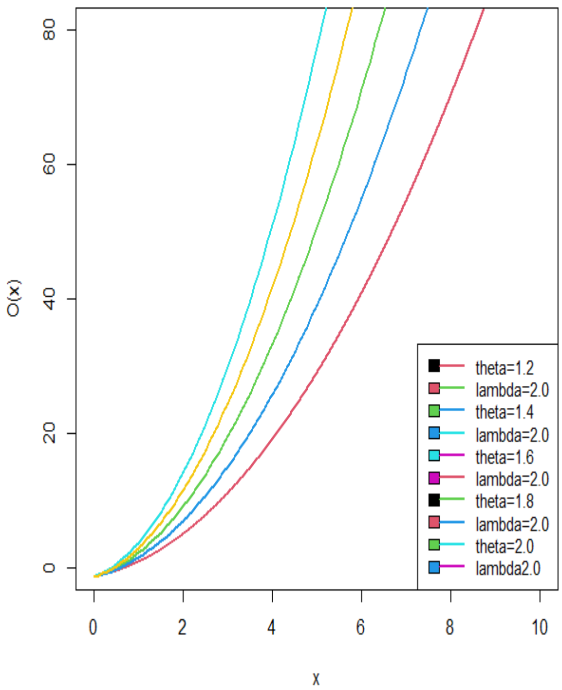


Figure.7 Odds rate plot of the Mixture of gamma and shanker distribution

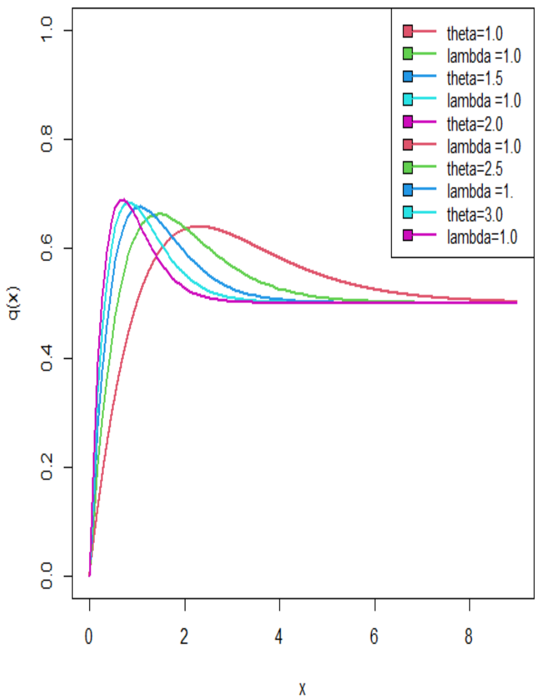
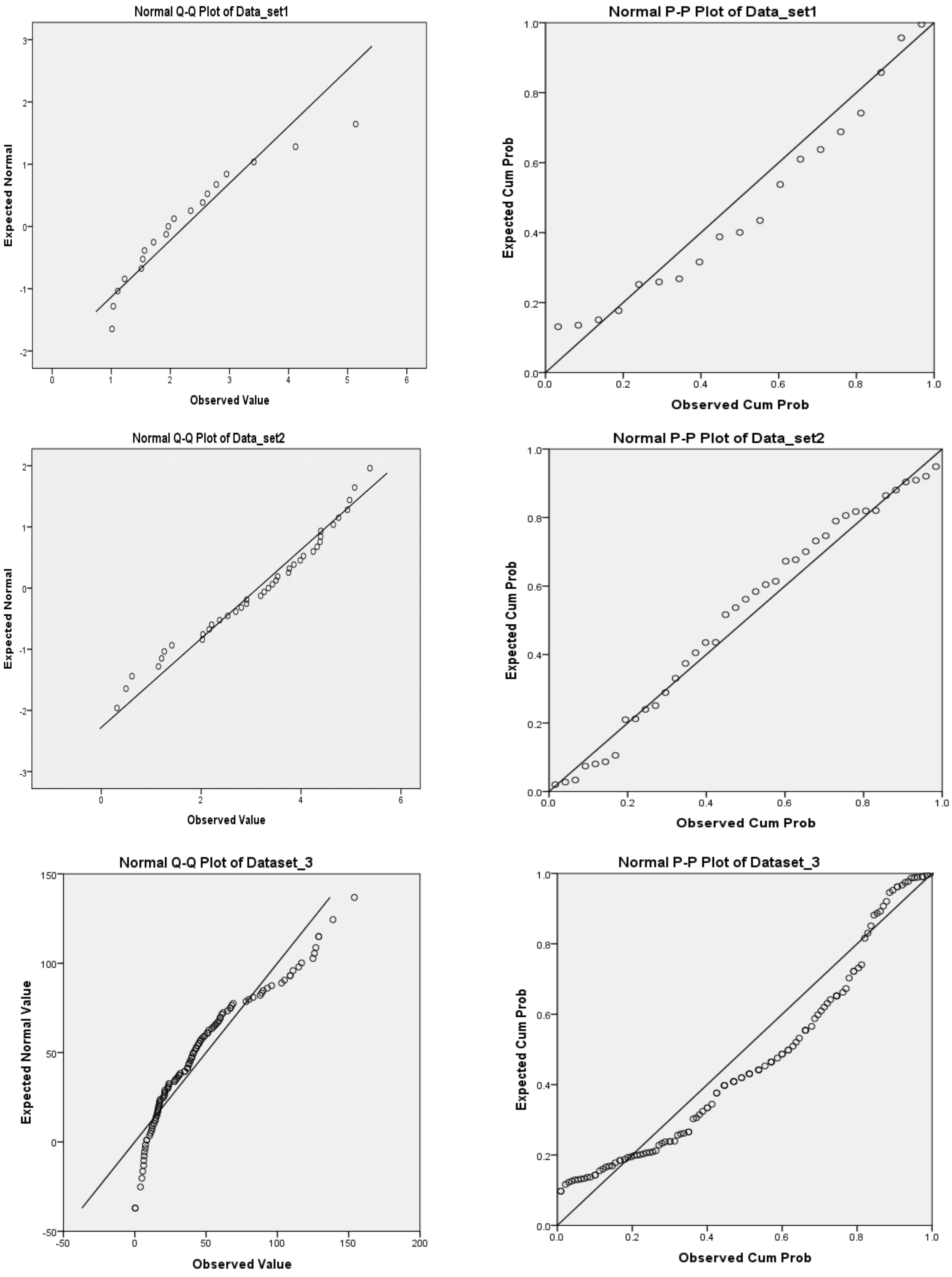


Figure.8 Quantile function plot of the Mixture of gamma and shanker distribution



Figures 9. The density, P-P, Q-Q Plots of the (MSGD) with cancer datasets.

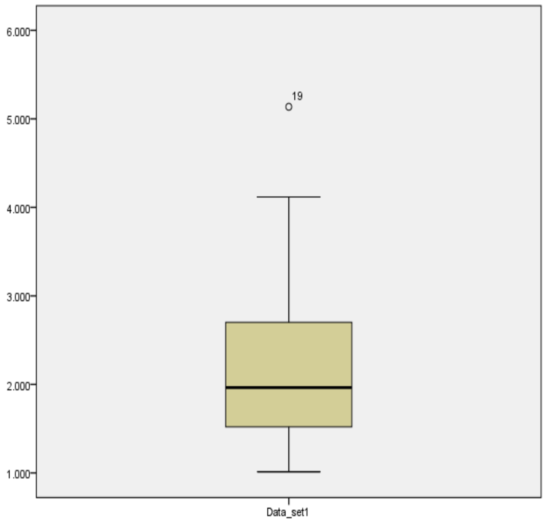


Figure 10. Boxplot for dataset 1

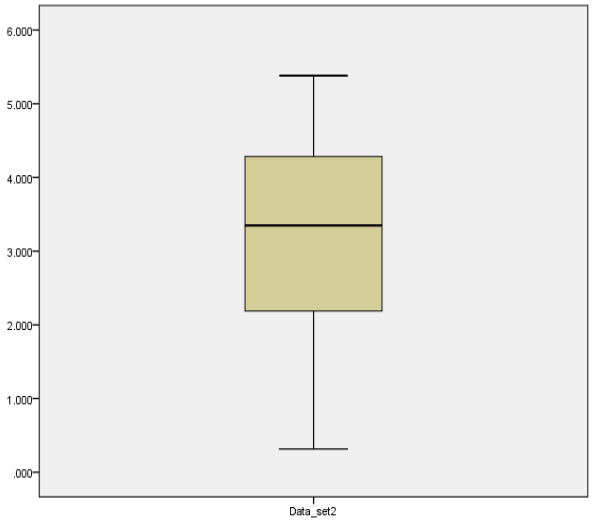


Figure 11. Boxplot for dataset 2

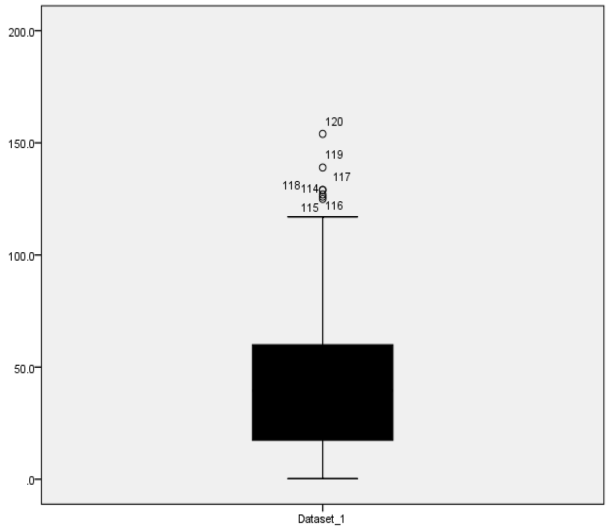


Figure 12. Boxplot for dataset 3.

14. Conclusion

In this paper, present a two-parameter distribution called an (MGSD). This is a blend of two well-known distributions: the gamma and the shanker distribution. Survival function and hazard function have been discussed. The statistical characteristics of the moments, the moments generating function, mean, variance, skewness, kurtosis, order statistics, stochastic ordering, entropies, Bonferroni and Lorenz curves, and the method of maximum likelihood estimation of the parameters. Moreover, the derived distribution is applied to three real data sets and compared with the other well-known distributions. Show that the blend of Gamma and Shanker Distributions provides a better fit than other well-known distributions.

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