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Application of Exponential and Logistic Models in Estimating the Population of Bulukumba Regency in 2020-2030

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ABSTRACT

Population data is useful as planning material in making various policies, including avoiding imbalances between the number of health facilities and services and the population in an area as well as other facilities such as schools, markets, and other public facilities. Ordinary differential equations of exponential and logistic models are used in modeling population dynamics in Bulukumba Regency to obtain population estimates until 2030. The determination of the future population of Bulukumba Regency is based on the growth rate and capacity obtained using the exponential and logistic approaches. The results obtained show that the estimation using the exponential model and the logistic model estimation for 2015-2019 are close to the data from the Central Bureau of Statistics. However, the logistic model is more accurate than the exponential model which is more significantly close to the data from BPS. So that the results of the logistic model are better than the exponential. The logistic model assumes that Bulukumba Regency has a capacity of K = 450000, while the exponential model assumes that the population increases exponentially.

Ordinary Differential Equation; Exponential Equation; Logistic Equation

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1. Introduction

The change in population every year is a form of growth. Population growth can affect the progress and prosperity of an area. Population increase is influenced by birth, death, and migration factors. The population in Bulukumba Regency has increased, therefore it is necessary to estimate the population in the future. Bulukumba Regency is located at the tip of Sulawesi island, precisely in South Sulawesi province. Bulukumba Regency is also famous for its beautiful beach tourism and the local government is also serious in advancing some of its tourism potential (Amin et al., 2022).

Mathematical models have been widely used in problems of disease spread that affect population dynamics in a country, (Hukmah et al., 2023; Madaki et al., 2023, 2020; Sulma et al., 2023), as well as analyzing the relationship between corruption cases and infectious diseases (Radja & Ndii, 2022; Wahid et al., 2023). Some mathematical models that study population growth are exponential and logistic growth models. These models are able to provide an overview of population changes and provide an estimate of the population size in the future.

The use of exponential and logistic growth models has been widely studied in various countries. The exponential growth model assumes that the population growth rate is proportional to the size of the population, if the growth rate is positive then the population will continue to grow exponentially and vice versa. The logistic growth model is the result of the refinement of the exponential model, assuming that at a certain time the population will approach the equilibrium point (Patel & Prajapati, 2016; Ullah et al., 2019; Wei et al., 2015).

In research (Kurniawan et al., 2017) examining the application of logistic differential equations to population growth in the city of Surabaya, it was found that the logistic model was more accurate in predicting population. Prediction of population with a logistic equation using the Adams-Bashforth-Moulton method in West Kalimantan Province in 2021 resulted in a MAPE value of 0.689% with very good criteria (Putri & Noviani, 2022). Based on research (Anggreini, 2020), it is found that with the exponential model the population continues to increase, while the logistic model will not exceed the carrying capacity of 43,997,165. In the research (Anggreini, 2018) it was found that the amount of carrying capacity that limits population growth in Tulungagung is 1,089,103 people with an intrinsic growth rate of $r = 0.07480$ with the estimated population in 2025 is 1.055.578 people.

Mathematical modelling is one way to represent complex problems into mathematical form. Mathematical models can be in the form of equations to represent the problem to be solved (Ndii, 2018).

2. Method

The Exponential Growth Model was introduced in 1798, Thomas Malthus created a basic population growth model known as the exponential growth model (Nuraeni, 2017). If it is assumed that $P(t)$ is a function derived with respect to time, thus a continuous function, then assume that $\frac{dP}{dt}$ expresses the rate of change of population proportional to the existing size. Then $\frac{dP}{dt} = kN$, where k is a proportional constant. In this model it is assumed that the population increases with a population growth rate proportional to the size of the population (Nuraeni, 2017). Suppose $P(t)$ states the population size at time t, and k states the population growth rate then the exponential population model is expressed in the form:

$$\frac{dP}{dt} = kP(t) \tag{1}$$

a special solution is obtained

$$P(t) = P_0 e^{kt} \tag{2}$$

A more realistic growth model is called the Logistic model. This model remains the same as the Maltus growth equation, except that the average growth in this case depends on the size of the population itself expressed in the form of:

$$\frac{dp}{dt} = r\left(1 - \frac{p}{K}\right)p \tag{3}$$

with $K = r/a$. Where r is the intrinsic growth rate and a is a positive constant (Waluya, 2006). The initial motivation for research on this model was that the natural growth model was not appropriate enough for a large enough population and limited space, causing problems due to population density that would reduce the population itself. This logistic population growth model is a refinement of the exponential growth model. In this model, the population size is influenced by the environment such as food supply. The logistic model assumes that at a certain time the population will approach the equilibrium point. At this point the number of births and deaths are considered equal so that the graph is close to constant. The solution of the logistic equation model is obtained by the variable separation method, namely (Boyce & DiPrima, 2009):

$$p = \frac{p_0 K}{p_0 + (K - p_0)e^{-rt}} \qquad (4)$$

with

p : Number of population at time t

p_0 : The number of initial population at time $t = 0$

K : Carrying capacity of an area for population

r : Rate of population growth rate

The research method used is quantitative using secondary data obtained from the Central Statistics Agency of Bulukumba Regency, in the form of quantitative data in the form of numbers. The first stage of the research is to determine the research subject, namely the population of Bulukumba Regency in 2015-2019. The second stage, collecting research data, obtained from secondary data from the Central Statistics Agency of Bulukumba Regency. The third stage is to analyze the data using exponential and logistic models and then determine the carrying capacity (K) and instrinsic growth rate (r) for the logistic model. The fifth stage, comparing the estimation results obtained with the results of the population census of Bulukumba Regency in 2015-2019. The last stage is to calculate the estimated population of Bulukumba Regency in the future from 2020 to 2030.

3. Result and Discussion

To obtain the results of population projections in Bulukumba Regency, first analyze the calculation of the population so that the pattern and amount of data to be used can be known. (Anggreini, 2020). The amount of data used in this study is based on the number of population projections available and accessible at the Central Statistics Agency of Bulukumba Regency. Based on the available data, it was found that the average population of Bulukumba Regency from 2015-2019 was 415.671 people, the highest population in 2019 was 420.603 people, and the lowest population in 2015 was 410.485 people. In Table 1 below, the population of Bulukumba Regency in 2015-2019 is given, which is obtained from data from the Central Statistics Agency of Bulukumba Regency.

Table 1. Total Population of Bulukumba Regency

Year	Total Population P(t)
2015	410.485
2016	413.229
2017	415.713
2018	418.326
2019	420.603

The data in Table 1. was used to calculate the estimated population of Bulukumba Regency with the 2015 population as P_0.

a. Population growth solution with exponential model

Using differential equations $P(t) = P_0 e^{kt}$, where $P_0 = 410.485$. To obtain projections, calculations in previous years were analyzed to see the trend direction of the data used. The value of k was obtained using the population numbers P_0 and $P(1)$ as follows:

$$P(1) = P_0 e^{k}$$

$$413.229 = P_0 e^{k} \tag{5}$$

for $t = 2$ is obtained

$$415713 = P_0 e^{2k} \tag{6}$$

from equations (5) and (6) obtained $k = 0{,}00599$. Thus the population for each year can be estimated by $P_0 = 410485$

$$P(t) = 410.485\ e^{0{,}00599t}$$

for $t = 3$, is obtained

$$P(3) = 410485\ e^{0{,}00599(3)} = 417.928$$

So, the estimated population of Bulukumba district with the exponential model is given in Table 2 below:

Table 2. Population of Bulukumba Regency with exponential model

Year	Total Population P(t)
2015	410.485
2016	412.951
2017	415.432
2018	417.928
2019	420.438
2020	422.965
2021	425.506
2022	428.062
2023	430.634
2024	433.222
2025	435.824
2026	438.442
2027	441.077
2028	443.727
2029	446.393
2030	449.075

Table 2 shows that the population always increases every year. Based on the exponential model, the population of Bulukumba Regency in 2030 is 449,075 people, meaning that since 2015 there has been an increase in population of 38,590 people. The error value of the estimation results can be calculated using data obtained from Central Statistics Agency of Bulukumba Regency and the estimation results obtained in 2015-2019, with a MAPE (Mean Absolute Percentage Error) value of (Anggreini, 2020):

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{|x_i - F_i|}{x_i} \times 100\% \tag{7}$$

with

n : Amount of data

x_i : Actual data

F_i : Prediction result data

The value generated through the calculation of MAPE, shows the forecasting criteria shown in Table 3.

Table 3. Indicator MAPE Value

MAPE Value	Forecasting Criteria
MAPE<10%	Highly Accurate
10%≤MAPE≤20%	Very Good
21%≤MAPE≤50%	Makes Sense
MAPE>51%	Inaccurate

By using equation (7), the MAPE value for the estimation results using the exponential model is as follows

$$MAPE = \frac{1}{5}\sum_{i=1}^{5}\frac{|x_i - F_i|}{x_i} \times 100\% = 0{,}0538\%$$

Based on the results of the MAPE value obtained, it shows that the forecasting results using the exponential model are included in the highly accurate category.

b. Population growth solution with logistic model

Population estimates using the logistic model are obtained by determining the population growth rate using a value of $p_0 = 410485$, $p(1) = 413.229$ and it is assumed that the capacity $K = 450.000$ so that the value is obtained $r = 0{,}078633387$. This value is substituted into the equation

$$p(t) = \frac{p_0 K}{p_0 + (K - p_0)e^{-rt}}$$

untuk nilai $t = 2$, diperoleh

$$p(2) = \frac{(410485)(45000)}{410485 + (450000 - 410485)e^{-(0{,}078633387)(2)}}$$

$$p(2) = 415.798$$

Thus, the estimation results in Table 4 are obtained for different values of t

Table 4. Population of Bulukumba Regency with logistic model

Year	Total Population P(t)
2015	410.485
2016	413.229
2017	415.798
2018	418.202
2019	420.449
2020	422.547
2021	424..505
2022	424..332
2023	428.034
2024	429.620
2025	431.097
2026	432.470

2027	433.748
2028	434.936
2029	436.039
2030	437.065

The calculation results in Table 4 are based on the values of $p_0 = 410.485$, $K = 450.000$, and $r = 0{,}078633387$ shows that the population is always increasing every year. In 2030 the population reached 437.065 people, meaning that since 2015 there has been an increase in population of 26.580 people. It can be seen that the total population increase using the exponential model is greater than the logistic model. This is because the exponential model assumes that the population grows under ideal conditions, without considering that an environment has limited resources. As for the logistic model, it assumes that a logistic factor, namely the carrying capacity of an area for population, is also taken into account. For the logistic model, a MAPE value of 0,0173% was obtained, including in the highly accurate category. The smallest MAPE value is the best value because the percentage error is the smallest, and the smallest value is obtained using the logistic model.

4. Conclusion

Based on the results obtained, it shows that the first-order differential equation using the logistic model and the exponential model in estimating the rate of change in population in Bulukumba Regency provides results that are quite close to the data from the Central Statistics Agency of Bulukumba Regency for 2015-2019. However, the logistic model is more accurate than the exponential model which is more significantly close to the data from BPS. So that the better estimation result for the total population of Bulukumba Regency in 2020-2023 is the estimation result obtained from the logistic model.

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